When you are done with your homework you should be able to…

π Understand the notation for a function of several variables
π Sketch the graph of a function of two variables
π Sketch level curves for a function of two variables
π Sketch level surfaces for a function of three variables

Warm-up: Find two functions such that the composition $h(x) = (f \circ g)(x) = \sin^2 x$

$f(x) =$ _____

$g(x) =$ _____

DEFINITION: A FUNCTION OF TWO VARIABLES

Let D be a set of ordered pairs of real numbers. If to each ordered pair (x, y) in D there corresponds a unique real number $f(x, y)$, then f is called a **function of x and y**. The set D is the **domain of f**, and the corresponding set of values for $f(x, y)$ is the **range** of f.

Example 1: Find and simplify the function values.
$$g(x,y) = \ln|x+y|$$

a. $g(2,3)$

b. $g(e,0)$

c. $g(0,1)$

Example 2: Describe the domain and range of each function.

a. $f(x,y) = \arccos\left(\dfrac{y}{x}\right)$

b. $g(x,y) = x\sqrt{y}$

Example 3: Sketch the surface given by the function.

a. $g(x, y) = \left(\dfrac{1}{2}\right)x$

b. $z = \dfrac{1}{2}\sqrt{x^2 + y^2}$

LEVEL CURVES

We can also visualize a function of two variables using a scalar field. This involves assigning a scalar value to z. This is then assigned to the point (x, y).

Example 4: Describe the level curves of the function. Sketch the level curves for the given *c*-values.

$$f(x, y) = \frac{x}{x^2 + y^2}, \quad c = \pm\frac{1}{2}, \ \pm1, \ \pm\frac{3}{2}, \ \pm2$$

Example 5: Sketch the graph of the level surface $f(x, y, z) = c$ at the given value of *c*.

$$f(x, y, z) = \sin x - z, \quad c = 0$$